

# MULTI-OBJECTIVE HYBRID OPTIMAL CONTROL FOR MULTIPLE-FLYBY INTERPLANETARY MISSION DESIGN USING CHEMICAL PROPULSION

Jacob A. Englander<sup>1</sup>, Matthew A. Vavrina<sup>2</sup>

## INTRODUCTION

Preliminary design of high-thrust interplanetary missions is a highly complex process. The mission designer must choose discrete parameters such as the number of flybys and the bodies at which those flybys are performed. For some missions, such as surveys of small bodies, the mission designer also contributes to target selection. In addition, real-valued decision variables, such as launch epoch, flight times, maneuver and flyby epochs, and flyby altitudes must be chosen. There are often many thousands of possible trajectories to be evaluated. The customer who commissions a trajectory design is not usually interested in a point solution, but rather the exploration of the trade space of trajectories between several different objective functions. This can be a very expensive process in terms of the number of human analyst hours required. An automated approach is therefore very desirable. This work presents such an approach by posing the impulsive mission design problem as a multi-objective hybrid optimal control problem. The method is demonstrated on several real-world problems.

Two assumptions are frequently made to simplify the modeling of an interplanetary high-thrust trajectory [1] during the preliminary design phase. The first assumption is that because the available thrust is high, any maneuvers performed by the spacecraft can be modeled as discrete changes in velocity. This assumption removes the need to integrate the equations of motion governing the motion of a spacecraft under thrust and allows the change in velocity to be modeled as an impulse and the expenditure of propellant to be modeled using the time-independent solution to Tsiolkovsky's rocket equation [1]. The second assumption is that the spacecraft moves primarily under the influence of the central body, *i.e.* the sun, and all other perturbing forces may be neglected in preliminary design. The path of the spacecraft may then be modeled as a series of conic sections. When a spacecraft performs a close approach to a planet, the central body switches from the sun to that planet and the trajectory is modeled as a hyperbola with respect to the planet. This is known as the method of patched conics [1]. The impulsive and patched-conic assumptions significantly simplify the preliminary design problem.

Many researchers have addressed the problem of finding the optimal high-thrust mission for a fixed destination and flyby sequence. Of particular relevance to this work are methods which employ stochastic global search methods such as genetic algorithm (GA)s, differential evolution (DE), particle swarm optimization (PSO), monotonic basin hopping (MBH), ant colony optimization (ACO), and inflationary differential evolution (IDEA) [2, 3, 4, 5, 6, 7]. These techniques, when coupled with an appropriate transcription [2, 4], are capable of finding globally optimal solutions to the fixed-sequence interplanetary mission design problem without requiring any *a priori* knowledge of the solution. However the works cited above only address the design problem for a fixed flyby sequence and destination and are therefore not sufficient to solve the full mission design problem.

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<sup>1</sup> Aerospace Engineer, Navigation and Mission Design Branch, NASA Goddard Space Flight Center, Greenbelt, MD, 20771, USA, Member AIAA

<sup>2</sup> Senior Systems Engineer, a.i. solutions, Inc., 10001 Derekwood Ln. Suite 215, Lanham, MD, 20706, USA

The traditional method to search over a wide space of possible destinations and/or flyby sequences is to grid over candidate sequences, launch epochs, and planet-to-planet flight times and represent the trajectory between each pair of bodies with either a Lambert arc [8] or an arc which includes a deep-space maneuver (DSM) [9]. Alternatively, one can design the mission one body-to-body phase at a time using a graphical approach involving Pork Chop Contour (PCC) and orbital resonance plots [10]. However as the size of the design space increases, *i.e.* as flybys, destinations, or DSMs are added, grid searches become more and more expensive. Other techniques which may be more efficient than a grid search also exist. Gad and Abdelkhalik [11, 12] solve the multiple-flyby problem using a GA with mixed-integer programming. Another method, by Vasile and Campagnola [13], uses a set of successive deterministic algorithms to find candidate low-thrust, multiple flyby trajectories.

Another approach is to formulate the interplanetary design problem as a hybrid optimal control problem (HOCP). A HOCP is an optimization problem that is composed of two separable sub-problems, one with discrete variables and the other with continuous variables [14, 15]. For interplanetary design, the first problem is to choose the discrete parameters that define the mission, such as number of flybys, choice of flyby bodies, and, for some types of missions, the destination. The second problem is to find the time history of control variables, such as launch date, flight times, thrust magnitude and direction, flyby altitudes, and encounter velocity vectors that characterize the optimal trajectory for each set of discrete parameters. A HOCP can be solved using two nested optimization loops. The “outer-loop” solves the integer programming problem defining the discrete parameters. Each candidate solution to the “outer-loop” problem defines an “inner-loop” trajectory optimization problem. This approach was demonstrated first by Chilan, Wall, and Conway [16] for trajectories without flybys and then by Englander, Conway, and Williams for trajectories that include flybys and either impulsive chemical propulsion [17] or low-thrust electric propulsion [18]. All of these methods used a GA to solve the outer-loop problem and a variety of stochastic global search algorithms to solve the inner-loop problem.

However, all of the methods above find only a single “optimal” trajectory, that is, optimal according to a single objective function. Preliminary mission design requires the exploration of a multi-objective trade space. The designer must find not a single solution but instead the Pareto front, surface, or hyper-surface (depending on the number of objectives) between several objective functions. Several researchers have addressed such problems in the past for problems with a fixed flyby sequence and fixed destination. Coverstone-Carroll, Hartmann, and Mason [19] used a multi-objective GA with an indirect trajectory optimizer. Vavrina and Howell [20] also used a multi-objective GA hybridized with a direct trajectory optimization method. Both research groups found non-dominated fronts of delivered mass versus flight time. In addition, Vasile and Zuiani [21] demonstrated a multi-objective algorithm for finding the non-dominated front between flight time and  $\Delta v$  for impulsive-thrust missions with fixed destination and flyby sequence. Most recently, Izzo *et al.* developed a multi-objective algorithm for finding the optimal Jovian capture trajectory given a fixed sequence of moon flybys [22].

In this work we present a new framework for multi-objective optimization of low-thrust interplanetary trajectories where the flyby sequence, and sometimes the destinations themselves, are not known *a priori*. The approach presented here is an extension of the HOCP technique for low-thrust trajectory and systems design previously introduced by these authors [23, 24]. The mission design problem is formulated as a HOCP where the outer-loop chooses the number of flybys, the identity of the flyby bodies, and, when appropriate, the destination. The outer-loop is based on the “null-gene” transcription presented by Englander, Conway, and Williams [17], a “cap and optimize” approach for varying the flight time and launch date, and the NonDominated Sorting Genetic Algorithm II (NSGA-II) multi-objective GA developed by Deb [25]. The innerloop is based on a modified version of the multiple gravity assist with deep-space maneuver (MGADSM) transcription described by Vinko and Izzo [4] which is constructed to interface with the stochastic global search algorithm MBH [26, 27, 7].

The proposed technique is demonstrated on several example problems, including a reproduction of the Cassini mission, a study of trajectory and alternate target opportunities for the OSIRIS-REx mission, and a notional mission to Jupiter in the 2020s. Figure 1 below shows a representative set of the non-dominated optimal trajectories from the Jupiter example, composed of missions which are optimal in launch epoch,

flight time, and delivered mass to an elliptical orbit about Jupiter. Figure 2 shows a sample trajectory from that set.

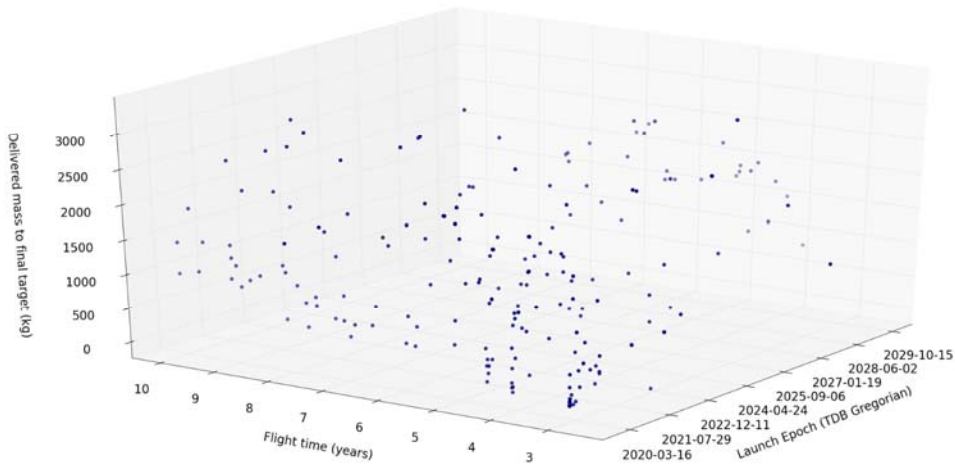


Figure 1: Non-Dominated Set of Optimal Trajectories for a Notional Mission to Jupiter in the 2020s

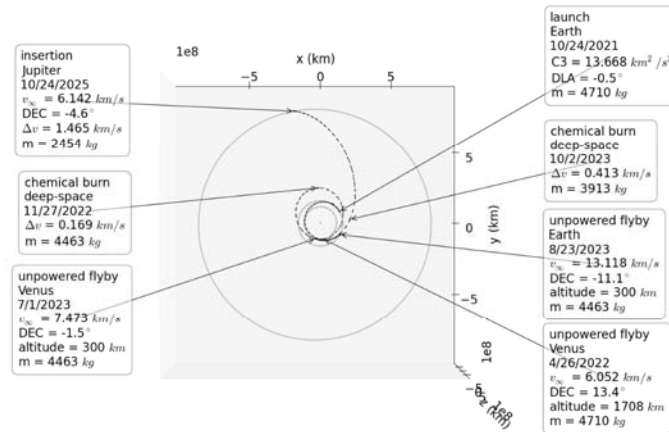


Figure 2: A Sample Trajectory from the Jupiter Example Problem

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